Modeling and analysis of the response of a triaxial, frequency-domain electromagnetic induction sensor to a buried linear conductor

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ABSTRACT

This paper presents analytical modeling results for a triaxial frequency-domain electromagnetic-induction (EMI) sensor over a homogeneous earth containing a long linear conductor. Although the conductor studied is intended to represent an underground wire or pipe, it can represent any subsurface, linear geologic structure that can channel current. Treating the sensor transmitter as a vertical magnetic dipole, the model combines the well-known solution for the magnetic field arising from the interaction with the earth with the solution for the induced magnetic field from the excited subsurface conductor. Expressions for the three components of the magnetic field at an arbitrary point above the earth are presented. Two types of coupled, moving transmitter-receiver configurations (coaxial and coplanar) were

INTRODUCTION

Active electromagnetic induction (EMI) sensors that historically have been used solely for geophysical investigations now are being adapted and used for applications such as locating unexploded ordnance (UXO) and land mines (Huang and Won, 2003b), underground tunnel detection (Mahrer and List, 1995), and archaeological studies (Osella et al., 2005). Understanding the output and capabilities of such sensors is a critical part of assessing their potential success. In this paper, we consider the problem of tunnel detection and present analytical modeling results for a triaxial, frequency-domain EMI sensor over a homogeneous earth containing a long, linear conductor. The conductor studied is intended to represent the wiring often found in underground tunnels (Mahrer and List, 1995). It also can represent underground infrastructure such as pipes or utility cables. Alternatively, it can represent any subsurface, linear geologic structure that can channel current (Tsubota, 1979; Tsubota and Wait, 1980).

considered, and the model is sufficiently flexible to allow for many other sensor variations to be studied. Characteristics of the sensor signals were explored through several parametric modeling studies that demonstrate the sensitivity of the signals to transmitter frequency, earth conductivity, conductor depth, sensor geometry, and crossing angle. Using simple relationships developed from analysis of the sensor signals, key parameters such as conductor depth and orientation can be estimated. The ability of the model to predict and characterize sensor output should prove helpful in distinguishing between geologic features and manmade underground infrastructure. These modeling results also are expected to facilitate frequency-domain EMI data analysis and interpretation, sensor design and operation, and the development of detection and classification algorithms.

The past several decades have seen a proliferation in the development of compact EMI sensors for near-surface investigations (Frischknecht et al., 1991; Won, 2003), yet there are few rigorous, analytical treatments of specific fielded sensors. Sensors are becoming increasingly more sophisticated and one key advancement is the use of multicomponent receivers (Kriegshäuser et al., 2000; Rosthal et al., 2003). Multicomponent sources and receivers have been studied extensively for borehole applications where they have shown their value in probing anisotropic formations (e.g., see Tompkins et al., 2004, and the references therein). Smith and Keating (1996) have illuminated the benefits of multicomponent sensors for airborne, time-domain EMI investigations. For this paper, we studied two types of coupled, moving source-receiver configurations, each with triaxial receivers (see Figure 1). We refer to the sensors as being coplanar (horizontal coils in same horizontal plane) or coaxial (horizontal coils along same vertical axis). The coplanar configuration is a variant of the classic Slingram geometry with the transmitter and receiver coils in a rigid configuration along a boom. The Geonics

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EM31 (Geonics Ltd., note TN-06) is an example of this configuration and has been widely used as a ground conductivity meter. The coaxial configuration is also a rigid arrangement and operates in a gradiometric mode, measuring the difference between receivers. The Geophex GEM-5 (Geophex, 2009), an example of this configuration, is designed for tunnel, UXO, and utility detection. Other variations of these geometries exist, e.g., Stolarczyk et al. (2005), which is a horizontal gradiometer configuration. In most cases, the sensors are used as surface-profiling instruments, measuring response variations along the survey path. To yield depth-sounding information, sensors can employ multiple frequencies, multiple receivers at different offsets from the transmitter, or some combination of the two.

Here our focus is to understand and characterize the expected output of such sensors and to illustrate how that insight can lead to improved data interpretation and analysis. For that purpose, we modeled the total field as it would be sensed by a receiver coil at an arbitrary location above the earth, i.e., the combination of the primary field, the field resulting from the presence of the earth, and the field from the buried linear conductor. The model is based on an integral transform approach for a vertical-magnetic-dipole source and a triaxial receiver that measures the magnetic field in three orthogonal directions. The complete solution was obtained by solving two separate problems: (1) the fields resulting from a vertical-magnetic-dipole source located above a uniform earth and (2) the fields resulting from a buried linear conductor excited by a vertical magnetic dipole above a homogeneous earth. The former problem has been well studied, e.g., Wait (1955), Keller and Frischknecht (1966), Ward and Hohmann (1987), among others. The latter problem for an excited linear conductor in the earth has been solved analytically by a number of others. Howard (1972) studies the case of a cylindrical inhomogeneity in a uniform half-space with line source excitation. Wait and Umashankar (1978) solve the problem for a cable excited by a current point source in homogeneous and layered media. Watts (1978) examines the situation of a buried wire in a layered earth for planewave excitation. Hill (1988) considers a dipole source and an infinitely long conductor within a uniform whole-space. Tsubota and Wait (1980) look at the problem of a vertical-magnetic-dipole source



Figure 1. Sensor geometries studied. Each consists of a single vertical-magnetic-dipole transmitter (TX) and (a) a single triaxial receiver (RX) and (b) two triaxial receivers used in a gradiometric mode.

over a two-layer earth containing a long linear conductor; however, only the vertical component of the magnetic field resulting from the conductor is presented explicitly. In addition, Tsubota and Wait show only computed results for the transient response of the induced current in the conductor. They do not provide frequency-domain profiling results that are of interest for the types of sensors considered here. Here we follow the approach of Tsubota and Wait for the simplified case of a homogeneous earth. As an extension of their results, we provide the expressions for all three components of the magnetic field resulting from the conductor. We extend their work further by using these expressions to predict and analyze the frequency-domain responses resulting from a buried linear conductor for two EMI sensor configurations.

We have implemented the model in the MATLAB computing environment and used it to explore details of the anticipated sensor signals. We performed several parametric modeling studies to illustrate the capabilities of the model and show how the sensor signals will depend on variables such as transmitter frequency, earth conductivity, conductor depth, sensor geometry, and crossing angle. Several sensor response curves are presented and suggest that many of these dependencies can be complex. Using simple relationships derived from analysis of the sensor signals, we demonstrate how important parameters such as conductor depth and orientation can be estimated. Ways in which the response curves and the derived relationships can facilitate data analysis and interpretation, sensor design and operation, and the development of detection and classification algorithms are discussed.

ANALYTICAL MODELING

Problem definition and assumptions

For this study, we considered the case of a single horizontal transmitter coil and a single receiver consisting of a triaxial set of coils (refer to Figure 2). The coil dimensions were assumed to be small compared to the measurement distance (i.e., the transmitter-receiver



Figure 2. EMI sensor transmitter-receiver geometry with buried, linear conductor. The transmitter (TX) is a horizontal coil with axis in the z-direction. The receiver (RX) is made up of a horizontal coil with axis in the z-direction, a vertical coil with axis in the y-direction, and a second vertical coil with axis in the x-direction. The conductor, which is infinite in length, runs directly below and parallel to the y-axis at a depth D. The polar coordinates ρ and ϕ measure in the xy-plane.

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separation) such that the transmitter was modeled as a magnetic dipole. For land-based mobile sensors, coil diameters are on the order of tenths of a meter to a couple meters (e.g., Geonics Ltd., note TN-06; Won et al., 1996; Won et al., 1997; Stolarczyk et al., 2005). For our work, we took the nominal separation distance to be s = 4 m. The transmitter coil was represented by a z-directed magnetic dipole of moment IA located at (x_d, y_d, z_d) . The three receiver coils were colocated at (x, y, z). The model formulation is sufficiently general such that the positions of the transmitter and each receiver coil can be arbitrary and many parameters can be varied. This permits the study of a wide range of sensor configurations, from those with moving transmitter and receiver(s) to those with a fixed transmitter and roving receiver(s). Land-based sensors and airborne sensors are equally treatable with the model. The air, region 0 (z > 0), was considered a free-space with conductivity $\sigma_0 = 0$, permittivity ε_0 , permeability μ_0 , and wavenumber $k_0 = \omega(\mu_0 \varepsilon_0)^{1/2}$. The ground, region 1 ($z \le 0$), was modeled as a homogeneous, lossy half-space, with conductivity σ_1 , permittivity $\varepsilon_1 = 10\varepsilon_0$, permeability $\mu_1 = \mu_0$, and wavenumber $k_1 = (\omega^2 \mu_1 \varepsilon_1 - i \omega \mu_1 \sigma_1)^{1/2}$. An implicit $e^{i\omega t}$ time dependence was assumed throughout, where $\omega = 2\pi f$ is the radial frequency and f is the transmitter frequency. See Table 1 for a complete listing of all properties and parameters used. When presenting the final model expressions, we assume quasi-static conditions because all relevant dimensions of the problem are small compared to the wavelengths considered. (The full expressions can be found in the appendices.) Lastly, given the relatively low frequencies involved, we ignored any interactions between induced currents in the earth and the current in the conductor (Kaufman and Eaton, 2001).

The magnetic field measured at the receiver was assumed to be composed of the following:

- Fields resulting from the dipole source: $\mathbf{H}^d = \mathbf{H}^{d,p} + \mathbf{H}^{d,s}$ (Wait, 1955), where the primary field is denoted with the superscript "*p*" and the secondary field is denoted with an "*s*". The primary field is the field in the absence of the conducting earth. The secondary field is that which results from the presence of the conducting earth.
- Field resulting from the presence of a long, linear subsurface conductor: H^c.

We assumed that the total magnetic field measured at the receiver is given by a superposition of the dipole fields and the conductor field,

$$\mathbf{H} = \mathbf{H}^d + \mathbf{H}^c = \mathbf{H}^{d,p} + \mathbf{H}^{d,s} + \mathbf{H}^c.$$
(1)

Our aim was to determine the three components of the total magnetic field **H** at the receiver location (x,y,z) in the air (z > 0).

Vertical magnetic dipole over a homogeneous, conducting earth

In this section, we summarize the results for a vertical-magneticdipole source over a homogeneous, conducting earth. Details of the solution can be found in Appendix A. The total magnetic field components resulting from the dipole source are written as the sum of the primary field and the secondary field. In Cartesian coordinates,

$$H_{x}^{d} = H_{x}^{d,p} + H_{x}^{d,s},$$
 (2)

$$H_{y}^{d} = H_{y}^{d,p} + H_{y}^{d,s},$$
 (3)

and

$$H_{z}^{d} = H_{z}^{d,p} + H_{z}^{d,s}.$$
 (4)

The primary fields at a receiver location (x,y,z) with $z_d = h$ are

$$H_x^{d,p} = \frac{3M(x-x_d)(z-h)}{[r_d^2 + (z-h)^2]^{5/2}},$$
(5)

$$H_{y}^{d,p} = \frac{3M(y - y_{d})(z - h)}{[r_{d}^{2} + (z - h)^{2}]^{5/2}},$$
(6)

and

$$H_z^{d,p} = \frac{3M(z-h)^2}{[r_d^2 + (z-h)^2]^{5/2}} - \frac{M}{[r_d^2 + (z-h)^2]^{3/2}},$$
 (7)

where $M = IA/4\pi$ and $r_d^2 = (x - x_d)^2 + (y - y_d)^2$. The secondary fields at a receiver location (x, y, z) with z > 0 are

$$H_x^{d,s} = M \frac{(x-x_d)}{r_d} \int_0^\infty \frac{\lambda - u_1}{\lambda + u_1} e^{-\lambda(z+h)} J_1(\lambda r_d) \lambda^2 d\lambda, \quad (8)$$

Table 1. Nominal properties and parameters

Parameter	Symbol	Value	Units
Free-space permeability	μ_0	1.2566×10^{-6}	W/Am
Free-space permittivity	$\boldsymbol{\varepsilon}_0$	8.85421×10 ⁻¹²	F/Am
Air conductivity	σ_{0}	0	mS/m
Air permeability	μ_0	μ_0	W/Am
Air permittivity	$\boldsymbol{\varepsilon}_0$	$\boldsymbol{\varepsilon}_0$	F/m
Earth conductivity	σ_1	100	mS/m
Earth permeability	μ_1	μ_0	W/Am
Earth permittivity	$\boldsymbol{\varepsilon}_1$	$10\varepsilon_0$	F/m
Transmitter frequency	f	10	kHz
Horizontal RX-TX separation	S	4	m
Vertical RX-TX separation	b	0.75	m
Sensor height (ground to TX)	h	1	m
Sensor azimuth w.r.t. <i>x</i> -axis	α	0	0
Source moment	IA	1000	Am ²
Conductor conductivity	$\sigma_{\scriptscriptstyle c}$	5.7×10^{7}	S/m
Conductor permeability	μ_c	μ_0	W/Am
Conductor permittivity	$\boldsymbol{\varepsilon}_{c}$	0	F/m
Conductor radius	а	0.005	m
Conductor depth	D	3	m

$$H_{y}^{d,s} = M \frac{(y - y_d)}{r_d} \int_{0}^{\infty} \frac{\lambda - u_1}{\lambda + u_1} e^{-\lambda(z+h)} J_1(\lambda r_d) \lambda^2 d\lambda, \quad (9)$$

and

$$H_z^{d,s} = M \int_0^{\infty} \frac{\lambda - u_1}{\lambda + u_1} e^{-\lambda(z+h)} J_0(\lambda r_d) \lambda^2 d\lambda, \qquad (10)$$

where $u_1^2 = \lambda^2 - k_1^2$ and J_0 and J_1 are Bessel functions of the first kind.

Vertical magnetic dipole over a homogeneous, conducting earth with a buried linear conductor

The geometry considered is shown in Figure 2. The core problem definition is unchanged from the previous section. The conductor is infinite in length and has radius *a*, conductivity σ_c , permittivity ε_c , permeability μ_c , and wavenumber $k_c = (-i\omega\mu_c\sigma_c)^{1/2}$. Refer to Appendix B for the details of the solution.

The magnetic fields resulting from the conductor as measured by a receiver located at (x,y,z) with z > 0 are

$$H_{j}^{c} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{I}(\beta) F_{j} \frac{\lambda}{u_{0}(u_{1}+u_{0})} \\ \times e^{-i(\lambda x+\beta y)} e^{-(u_{1}D+u_{0}z)} d\lambda d\beta,$$
(11)

where

$$F_{j} = \begin{cases} \lambda & j = x \\ \beta & j = y \\ -iu_{0} & j = z \end{cases}$$
(12)

 $u_n = (\beta^2 + \lambda^2 - k_n^2)^{1/2}$, n = 0, 1, and $\hat{I}(\beta)$ is the transform of the conductor current I(y). The transform of the current is



Figure 3. Plan view of survey geometry. The *xyz*-frame is fixed, and the x'y'z'-frame is a moving frame that is aligned with each sensor as shown. The angle the sensor frame makes with the *x*-axis is α .

$$\hat{I}(\beta) = \frac{(\hat{E}_y^d)_c}{Z_i + Z_e(\beta)}.$$
(13)

The term Z_i represents the internal impedance of the conductor,

$$Z_i = \frac{1}{2\pi a} \left(\frac{i\mu_c \omega}{\sigma_c}\right)^{1/2} \frac{I_0(ik_c a)}{I_1(ik_c a)},\tag{14}$$

where I_0 and I_1 are modified Bessel functions of the first kind. The term $Z_e(\beta)$ in equation 13 corresponds to an external impedance resulting from the surrounding earth,

$$Z_{e}(\beta) = \frac{\beta^{2} - k_{1}^{2}}{2\pi\sigma_{1}'} \Biggl\{ K_{0}[(\beta^{2} - k_{1}^{2})^{1/2}a] - K_{0}[2D(\beta^{2} - k_{1}^{2})^{1/2}] + \frac{k_{1}^{2}}{k_{1}^{2} - \beta^{2}} \int_{-\infty}^{\infty} \left(\frac{1}{u_{0} + u_{1}} - \frac{\beta^{2}}{k_{1}^{2}u_{0}}\right) e^{-2u_{1}D} d\lambda \Biggr\}, \quad (15)$$

where K_0 is the modified Bessel function of the second kind. The term $(\hat{E}_y^d)_c$ in equation 13 represents the transform of the *y*-component of the electric field resulting from the dipole source evaluated at the conductor and is given by

$$(\hat{E}_{y}^{d})_{c} = \frac{\mu_{0}\omega IA}{4\pi^{2}} \int_{-\infty}^{\infty} \frac{\lambda}{u_{0} + u_{1}} e^{i(\lambda x_{d} + \beta y_{d})} e^{-(u_{0}h + u_{1}D)} d\lambda.$$
(16)

Survey geometry

To describe the signals as recorded by a sensor in the field, we attached a second Cartesian frame that is aligned with the sensors as shown in Figure 3. This second frame is only relevant for the conductor-generated fields. The primary field and the field resulting from the earth are unaffected by sensor orientation because the geometry of the transmitter and receivers is fixed and the earth was assumed homogeneous. For the coplanar sensor, $H_x^{d,p} = H_y^{d,p} = 0$ because the transmitter and receiver are at the same height, and $H_z^{d,p}$ is nonzero. The secondary fields $H_x^{d,s}$ and $H_z^{d,s}$ are nonzero, and $H_y^{d,s} = 0$ because the transmitter and receiver both lie along the x'-axis. For the coaxial sensor, $H_x^{d,p} = H_y^{d,p} = 0$ because the transmitter and receivers are along the same vertical axis, and $H_z^{d,p}$ is nonzero. The secondary fields $H_x^{d,s}$ and $H_y^{d,s}$ are zero because the transmitter and receivers are vertically coaxial, and $H_z^{d,s}$ is nonzero. When the sensor crosses a conductor at an angle α as shown in the figure, the components of the conductor field in the sensor frame are:

$$H_{x'}^c = H_x^c \cos \alpha + H_y^c \sin \alpha, \qquad (17)$$

$$H_{v'}^{c} = -H_{x}^{c}\sin\alpha + H_{v}^{c}\cos\alpha, \qquad (18)$$

and

$$H_{z'}^{c} = H_{z}^{c}.$$
 (19)

All surveys are straight-line profiles. In each profile, the origin of the moving x'y'z'-frame is located directly over the conductor at an along-profile distance of 25 m.

RESULTS

Sensor signals

How one defines the sensor signal or interpretive parameter is somewhat arbitrary. Fundamentally, for each receiver coil, the sensor measures a continuous time-series of voltage at the coil. Then these signals are convolved with sine and cosine signals, either in circuitry or digitally, to extract the inphase (real) and quadrature (imaginary) components. Thus, a pair of signals is produced by each coil, for a total of six signals per triaxial receiver. In some cases, these pairs are combined to yield magnitude and phase information. Much of the analysis and interpretation of such sensors has focused on the measured inphase and quadrature components to yield maps of magnetic susceptibility and apparent conductivity, respectively. Huang and Won (2003a) conclude that apparent conductivity is one of the more promising quantities when subsurface anomaly detection is the goal. We based our definition of the coplanar sensor signals on measuring the normalized inphase and quadrature components of the secondary field in parts per thousand (ppt) of the primary field at the receiver:

$$\mathbf{S} = \frac{\mathbf{H} - \mathbf{H}^{d,p}}{H_z^{d,p}}.$$
 (20)

This definition resembles those in practice (Ward and Hohmann, 1987). It is necessary to remove the primary field from the measurement such that the secondary field resulting from an anomaly can be isolated. This can be done through hardware design as in sensors like the GEM-2 (bucking coil) and GEM-3 (magnetic cavity) (Won et al., 1996, 1997) or in system software using careful calibration. Furthermore, normalizing by the vertical component of the source field accounts for transmitter strength variations, whether intentional or unintentional. Because $H_x^{d,p} = H_y^{d,p} = 0$ at the coplanar receiver, the signals we considered are:

$$S_{x}^{R} = \operatorname{Re}[H_{x}/H_{z}^{d,p}] \quad S_{x}^{I} = \operatorname{Im}[H_{x}/H_{z}^{d,p}],$$

$$S_{y}^{R} = \operatorname{Re}[H_{y}/H_{z}^{d,p}] \quad S_{y}^{I} = \operatorname{Im}[H_{y}/H_{z}^{d,p}], \quad (21)$$

and

$$S_z^R = \text{Re}[(H_z - H_z^{d,p})/H_z^{d,p}] \quad S_z^I = \text{Im}[(H_z - H_z^{d,p})/H_z^{d,p}].$$

We based our definition of the coaxial sensor signals on measuring the normalized inphase and quadrature components of the difference between the receiver fields in parts per million (ppm) of the primary field at the upper receiver

$$\mathbf{S} = \frac{\mathbf{H}^1 - \mathbf{H}^2}{H^{d,p,1}},$$
(22)

where the numeric superscripts indicate the upper and lower receivers. We defined the following quantities to be our signals of interest:

$$S_x^R = \operatorname{Re}[(H_x^1 - H_x^2)/H_z^{d,p,1}] \quad S_x^I = \operatorname{Im}[(H_x^1 - H_x^2)/H_z^{d,p,1}],$$

$$S_y^R = \operatorname{Re}[(H_y^1 - H_y^2)/H_z^{d,p,1}] \quad S_y^I = \operatorname{Im}[(H_y^1 - H_y^2)/H_z^{d,p,1}],$$
(23)

and

$$S_z^R = \operatorname{Re}[(H_z^1 - H_z^2)/H_z^{d,p,1}] \quad S_z^I = \operatorname{Im}[(H_z^1 - H_z^2)/H_z^{d,p,1}].$$

All signals are in the sensor coordinate frame (the primes have been omitted).

To quantitatively characterize the signals for each receiver coil, we examined the peak-to-peak separation *P* (see Figure 4 inset), the signal width *W*, the signal dynamic range (DR), the signal-to-back-ground ratio (S/B), and for cases where $\alpha \neq 0$, the ratios of $S_y^{R,I}$ to $S_x^{R,I}$. The background was defined as the signal measured when no conductor is present. The dynamic range of signal *S* was defined as $|\max(S) - \min(S)|$. The signal width was defined as the greatest extent of the signal that deviates from the background by more than 10% of the signal dynamic range.

Apparent conductivity

Before exploring the signals defined above, we look briefly at the traditional apparent conductivity measurement. For the coplanar sensor arrangement, the last signal in expression 21 is related to an apparent conductivity by (Geonics Ltd., note TN-06)

$$\sigma_a = \frac{4}{\omega\mu_0 s^2} S_z^I.$$
 (24)

This expression is only valid for the low induction number range and for receiver coils at the air-earth interface. A correction can be applied to equation 24 to account for a sensor at a height h above the ground. Using Geonics Ltd., note TN-06,

$$\sigma_a = \frac{4}{\omega\mu_0 s^2} [4(h/s)^2 + 1]^{1/2} S_z^I.$$
 (25)

Figure 4 shows a survey profile of apparent conductivity from a fielded coplanar sensor. One of several anomalies is highlighted showing its shape in greater detail. Accompanying the actual signal is a closely matching model result using equation 25 for a thin buried conductor 2 m deep in a 55-mS/m earth. The model signal captures the magnitude and shape of the anomaly well. This result illustrates how buried linear conductors can contribute significantly to cultural noise in near-surface EMI measurements.



Figure 4. Sample apparent conductivity measurement from a fielded coplanar sensor. The highlighted anomaly resembles that of a buried, linear conductor with a peak-to-peak separation of P. The gray curve represents a corresponding model result for a thin, buried conductor at a depth of 2 m in a 55-mS/m earth.

Tunnel-conductor example

Highlighting our focus on the response of EMI sensors to subsurface, linear conductors such as wires and cables, Figure 5 shows the six model signals for a hypothetical tunnel conductor for both sensor types. For the coplanar sensor, the real and imaginary signals are dominated by a trough for the x-coil and a peak for the y-coil. In each case, the primary trough (peak) occurs when the receiver passes over the conductor. For the z-coil, both signals take on a symmetric peaktrough-peak shape. The central trough occurs when the sensor midpoint is directly over the conductor. The signals for the coaxial sensor are somewhat different from the coplanar sensor. For the coaxial sensor, the signals take on a peak-trough shape for the x-coil and a trough-peak shape for the y-coil. All x- and y-signals pass through zero directly over the conductor. The z-coil takes on a peak-troughpeak shape much like that seen for the coplanar sensor, but with a much larger difference between the real and imaginary magnitudes. For both sensors, there is content in the y-signal because $\alpha \neq 0$. For the coaxial sensor, the x- and y-components are precisely antisymmetric about the conductor because of the symmetry of the sensor geometry. This is not the case for the coplanar sensor because of its offset geometry. This example illustrates the types of signals that can be expected from a fielded sensor and how the signal shape can depend on the particular sensor configuration.



Figure 5. Model signals for a tunnel-conductor example. Coplanar sensor results on the left in ppt, coaxial sensor on the right in ppm. Dotted line indicates the along-profile location of the conductor. Parameters are: $\sigma_1 = 10 \text{ mS/m}$, f = 100 kHz, s = 4 m (coplanar), b = 0.75 m (coaxial), $\alpha = 30^\circ$, and D = 3 m.

Signal characteristics

The model can be used to explore how the sensor signals depend on various variables, some of which are controllable (e.g., transmitter frequency) and others that are not (e.g., conductor depth). A thorough investigation of the effects of all these parameters is beyond our scope here — the permutations of sensor configurations and conductor scenarios are practically endless. Instead, we present selected examples of results that illustrate the type of information that the model can provide and how that information can be used to improve data interpretation, sensor design, and detection algorithm development. In our analysis, we assumed a few parameters to be fixed. Specifically, we held constant all air properties, all permittivities and permeabilities, the sensor height, and the conductor radius and conductivity.

Dependence on conductor depth

We studied the effect of conductor depth on the sensor signals for three transmitter frequencies. Figure 6 shows how the along-profile signals for both sensors evolve as the depth is increased for a frequency of 100 kHz. As expected, a clear effect of depth on all signals is attenuation. Signal-to-background ratio falls off universally, as does dynamic range. It also is evident that the shape of the signal depends on the depth in a complex way. As will be seen, this complex dependency is true for the other parameters of the problem as well. Although the inphase signals for both sensors produce peak-troughpeak responses for all depths, the quadrature signal is more interesting. For shallow depths, the signals exhibit multiple extrema with two central peaks symmetric about the conductor. As the depth increases, the signals transition to a single central peak with troughs on either side. This type of behavior has been observed by others, e.g., Das et al. (1990). The signals also widen in general, although changes in the shapes of the signals make this a more subtle effect.

Figure 7 shows the coaxial signal peak-to-peak separation as a function of conductor depth for the three frequencies studied. The lower-frequency signals behave near-linearly with depth. Using



Figure 6. Evolution of sensor vertical signals (background removed) for increasing conductor depth. Coplanar sensor results on the left in ppt, coaxial sensor on the right in ppm. Curves progress from D = 1.5 m (black) to D = 6 m (light gray) in 0.5 m-increments. Parameters are: $\sigma_1 = 100$ mS/m, f = 100 kHz, s = 4 m (coplanar), b = 0.75 m (coaxial), $\alpha = 0^\circ$.

these signals, a rough prediction of the conductor depth is $D \approx 2(P - 1)/3$. The higher-frequency signals behave differently, in particular S'_z , which exhibits a reduction in *P* for depths greater than 1.5 m. This is because of the signal changing from having multiple extrema to having only a single central peak.

The signal-to-background ratio for the coplanar sensor is shown in Figure 8 as a function of conductor depth and transmitter frequency. The significant decay with depth is as expected. We also observe the decay with increasing frequency, which was anticipated as well. Although the S/B is helpful in gauging the relative magnitude of the signal, it is not the best measure for assessing detection performance; a signal-to-noise ratio (S/N) is more appropriate. For example, if we assume the noise to be 10% of the background value, then S/N = 10 S/B and the S/N for the higher-frequency signals would be near unity for conductor depths of about 3–4 m.



Figure 7. Peak-to-peak separation for S_z^R and S_z^I as a function of conductor depth and transmitter frequency for the coaxial sensor. At **A**, the signal transitions to a single, central peak (see Figure 6). Parameters are: $\sigma_1 = 100 \text{ mS/m}, b = 0.75 \text{ m}, \text{and } \alpha = 0^\circ$.



Figure 8. Signal-to-background ratio for S_z^R and S_z^I as a function of conductor depth and transmitter frequency for the coplanar sensor. At **A**, the signal transitions to a single, central peak (see Figure 6). Parameters are: $\sigma_1 = 100 \text{ mS/m}$, s = 4 m, $\alpha = 0^\circ$.

Dependence on survey azimuth

We studied the effect on the sensor signals of crossing the conductor at angles other than perpendicular for several sensor geometries. Figure 9 shows how the along-profile *x*- and *y*-signals for the coaxial sensor evolve as the survey azimuth is increased. Both *x*-signals begin as sharp peak-trough shapes. As the survey azimuth increases, these signals are reduced in amplitude and spread out slightly. The *y*-signals begin as null and as the azimuth is increased, evolve into smooth trough-peak responses.

The peak-to-peak separation for the coplanar vertical signals is shown in Figure 10 as a function of azimuth and transmitter-receiver spacing. There is little effect on P for angles less than 30°, after which the effect becomes more pronounced and the separation grows rapidly. The peak-to-peak separation also increases as the transmitter-receiver spacing is widened.



Figure 9. Evolution of coaxial sensor horizontal signals for increasing survey azimuth. Curves progress from $\alpha = 0^{\circ}$ (black) to $\alpha = 60^{\circ}$ (light gray) in 5° increments. Parameters are: $\sigma_1 = 100 \text{ mS/m}$, f = 10 kHz, b = 0.75 m, and D = 3 m.



Figure 10. Peak-to-peak separation for S_z^R and S_z^I as a function of survey azimuth and transmitter-receiver separation for the coplanar sensor. Parameters are: $\sigma_1 = 100 \text{ mS/m}$, f = 10 kHz, and D = 3 m.

The main impact of crossing the conductor at an angle is to introduce content into the *y*-signals. This is captured in Figure 11, which shows the ratio of the signal extrema for the horizontal signals. The coaxial sensor ratios follow the curve $\tan(\alpha)$ precisely for all values of transmitter-receiver separation. This is a result of the perfectly symmetric configuration of the coaxial sensor: regardless of the survey azimuth, the excitation of the conductor is the same relative to the receivers. The coplanar signals, on the other hand, deviate from $\tan(\alpha)$ slightly because of the sensor geometry and the position of the transmitter and receiver relative to the conductor for different crossing angles. The figure shows the nominal result for s = 4, and we find the coplanar curves approach $\tan(\alpha)$ for smaller values of *s*. The reason for this behavior is that the horizontally offset receiver of the coplanar sensor experiences a different conductor excitation as a



Figure 11. Ratio of $\max(|S_y^R|)$ to $\max(|S_x^R|)$ and $\max(|S_y^l|)$ to $\max(|S_x^l|)$, with mean offsets removed, as a function of survey azimuth for both sensor types. Parameters are: $\sigma_1 = 100 \text{ mS/m}$, f = 10 kHz, s = 4 m (coplanar), b = 0.75 m (coaxial), and D = 3 m.



Figure 12. Evolution of coplanar sensor signals (background removed) for increasing earth conductivity. Curves progress from σ_1 = 0.01 mS/m (black) to σ_1 = 500 mS/m (light gray) in near-logarithmic increments. Parameters: f = 10 kHz, s = 4 m, $\alpha = 0^\circ$, and D = 3 m.

function of azimuth. Because the deviation is minimal, a good estimate of the crossing angle is $\alpha = \tan^{-1}(R)$, where *R* represents one of the ratios considered in Figure 11.

Dependence on earth conductivity

We investigated the effect of varying the earth conductivity on the sensor signals. Figure 12 shows how the along-profile *x*- and *z*-signals for the coplanar sensor evolve as the earth conductivity is increased for three frequencies. The response changes are complex, particularly the quadrature signals. This complexity is witnessed by analyzing the signal widths, shown in Figure 13. The lower-frequency responses are relatively insensitive to earth conductivity until about 1-10 mS/m, at which point the signals become narrower. For the higher frequencies, the signals become narrower to a certain point and then begin to grow in width, followed by another period of narrowing. This behavior results from the signal changing shape dramatically as the conductivity increases. The discontinuities for the 100-kHz signals and the imaginary 10-kHz signal occur at points where the signal has changed shape sufficiently for the width to take on a different measure.

DISCUSSION

The modeling results of the previous section have practical utility in several areas. In the interpretation of EMI survey data from a triaxial (or single-axis) sensor, predicted signals like those shown in Figures 6, 9, and 12 can help distinguish between geologic features and man-made, underground infrastructure. Figure 4 provides a good illustration of such a situation. It has been long recognized that buried linear conductors can contribute to cultural noise (Watts, 1978), and that man-made conductors such as wires, pipes, and cables can profoundly impact electromagnetic probing of the subsurface (Wait and Umashankar, 1978). The predicted signals can also enable development of automated detection algorithms. For instance, the computed signals can be used as the deterministic component of a matched-filter technique that looks to find a known signal in additive noise. The nature of the signals also might encourage



Figure 13. Signal width for S_z^R and S_z^l as a function of earth conductivity and transmitter frequency for the coplanar sensor. Parameters: $s = 4 \text{ m}, \alpha = 0^\circ$, and D = 3 m.

particular approaches to detection, e.g., wavelet-based techniques (Benavides and Everett, 2005).

The modeling results can further facilitate EMI survey data interpretation through relationships developed from analysis of the signals themselves. For example, the peak-to-peak separation of the vertical signal (Figure 7) can be used to estimate conductor depth (Smith and Keating, 1996; Kelly, 1999). As noted, under certain conditions, a simple linear relationship between the peak-to-peak separation and the conductor depth might exist. In other cases, such as the high-frequency results of Figure 7, the relationship might be less straightforward but still quantifiable. Being able to estimate the conductor depth is of clear utility to anomaly localization and identification.

An equally important role for the model is in the design and evaluation of new, custom EMI sensors. For instance, results like those in Figure 8, along with knowledge of a detector's performance, can be used to predict the role of transmitter frequency on the effective depth of investigation for a specific set of conditions. By conducting various sensitivity analyses with the model parameters for different sensor configurations, a sensor design can be optimized for a particular purpose. This information also can be used to guide sensor operation in the field.

For existing sensors, the model can be used to plan surveys and experiments intelligently. Although the primary intent of showing the signal width as a function of earth conductivity in Figure 13 is to portray the potential complexity of the signal behavior, signal width has key implications for survey sampling as well. The combination of the signal width, the sensor profiling speed, and the sensor sample rate will determine if a given signal is sufficiently resolved. This can be especially critical for airborne applications, where sensor speed is significant.

A particular benefit of a triaxial receiver (over a single-axis receiver) is the ability to estimate conductor orientation from a single survey profile. With measurements of the horizontal components, their ratio can be used to estimate the conductor crossing angle (Figure 11). Smith and Keating (1996) have used a similar approach with their time-domain data to estimate the orientation of a conductive plate. The crossing angle also affects the peak-to-peak separation. Therefore, with an estimate of the crossing angle, the effect on the peak-to-peak separation can be accounted for and used to correct the estimate of the conductor depth. Multicomponent measurements are additionally useful because they provide more discriminating information for detection and characterization.

Although the model presented provides flexibility in terms of sensor configuration, there are a number of limitations that should be noted. The model is limited to a source that can be represented as a vertical magnetic dipole. Other sources such as electric dipoles, line sources, or magnetic dipoles with other orientations will require additional modeling. A number of the sensors in use today, however, can be represented using a vertical-magnetic-dipole source. The model also is limited by considering only a homogeneous earth. A straightforward extension would be a buried conductor in a stratified earth. Tsubota and Wait (1980) have considered such a situation for a two-layer model. Lastly, our modeling approach is only valid for conductors that are small in diameter relative to the other problem dimensions. In particular, the conductor diameter must be much smaller than the burial depth, $a \ll D$ (see Appendix B). For thin conductors such as wires or pipes, this does not pose too serious a limitation; however, for larger conductors such as broad, linear, conductive geologic features, this will require sufficient depth for the modeling approach to be valid. Also, conductor diameter must be much smaller than the skin depth, which is not expected to be an issue given the typically low frequencies used by EMI sensors of this type.

CONCLUSION

With traditional EMI sensors being adapted and used for nontraditional applications, geophysicists are having to recognize and interpret new types of anomalous signals. Subsurface infrastructure like wiring and piping, typically considered as cultural noise, is now considered as a target of interest to some. Underground tunnel detection is a clear example. Irrespective of how such infrastructure is regarded, it is important for proper data interpretation to be able to recognize the signals caused by such infrastructure. The modeling results presented here demonstrate the ability to predict the output signals of two particular EMI sensor configurations for a buried linear conductor. The configurations chosen reflect two fielded commercial sensors (the Geonics EM31 and the Geophex GEM-5), and with the flexibility of the model, many other sensor configurations can be studied.

The model we have outlined provides a useful framework for frequency-domain EMI sensor analysis. Through parametric studies, we have illustrated how sensitivities for a particular instrument can be readily determined. Results suggest that these sensitivities can be quite subtle and complex, requiring a solid understanding of the details of the near-field EMI physics. The model results can be used to improve data interpretation and analysis, sensor design and operation, and development of detection and classification algorithms. The modeling results also can serve to quantify the ability of existing and proposed sensors to meet survey requirements. Further work in this area will involve studying the responses to other buried conductors. Most importantly, we look to validate these model results with experimental data.

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APPENDIX A

VERTICAL MAGNETIC DIPOLE OVER A HOMOGENEOUS, CONDUCTING EARTH

In what follows, the reader is referred to Figure 2. We present the formulations for the primary magnetic field arising from a vertical-magnetic-dipole source and the ensuing secondary magnetic field resulting from the interaction with the conducting earth. We develop the general expressions first and then simplify the results when displacement currents are neglected.

PRIMARY MAGNETIC FIELD RESULTING FROM THE DIPOLE SOURCE

In the absence of the earth, the vertical magnetic dipole was represented using a single magnetic Hertz vector with only a *z*-component,

$$\Pi^{d} = (0, 0, \Pi^{d}_{7}), \tag{A-1}$$

where Π_z^d is given by

$$\Pi_z^d = \frac{IA}{4\pi} \frac{e^{-ik_0 R}}{R},\tag{A-2}$$

with $R^2 = (x - x_d)^2 + (y - y_d)^2 + (z - z_d)^2$ and $k_0 = \omega(\mu_0\varepsilon_0)^{1/2}$. Setting $z_d = h$ and $\rho_d^2 = (x - x_d)^2 + (y - y_d)^2$, then $R^2 = \rho_d^2 + (z - h)^2$. The primary magnetic field components are obtained from (Stratton, 1941)

$$\mathbf{H}^d = (\nabla \nabla \cdot + k_0^2) \mathbf{\Pi}^d. \tag{A-3}$$

Setting $M = IA/4\pi$, we have

$$H_x^d = M \frac{e^{-ik_0 R}}{R^5} (x - x_d)(z - h)(3 + 3ik_0 R - k_0^2 R^2), \quad (A-4)$$

$$H_{y}^{d} = M \frac{e^{-ik_{0}R}}{R^{5}} (y - y_{d})(z - h)(3 + 3ik_{0}R - k_{0}^{2}R^{2}), \quad (A-5)$$

and

$$H_{z}^{d} = M \frac{e^{-ik_{0}R}}{R} \Biggl\{ k_{0}^{2} \Biggl[1 - \frac{(z-h)^{2}}{R^{2}} \Biggr] + \Biggl(\frac{ik_{0}R + 1}{R^{2}} \Biggr) \\ \times \Biggl[\frac{3(z-h)^{2}}{R^{2}} - 1 \Biggr] \Biggr\}.$$
 (A-6)

If we neglect displacement currents $(k_0 \rightarrow 0)$, the primary field reduces to

$$H_x^d = \frac{3M(x - x_d)(z - h)}{R^5},$$
 (A-7)

$$H_{y}^{d} = \frac{3M(y - y_{d})(z - h)}{R^{5}},$$
 (A-8)

and

$$H_z^d = \frac{3M(z-h)^2}{R^5} - \frac{M}{R^3}.$$
 (A-9)

SECONDARY MAGNETIC FIELD RESULTING FROM THE DIPOLE-SOURCE INTERACTION WITH THE EARTH

To formulate the problem to include the earth, the primary field again was based on the potential in equation A-1. The secondary field was derived from magnetic potentials of the form $\psi(\lambda)e^{\pm(\lambda^2-k^2)^{1/2}z}J_0(\lambda\rho_d)$ (e.g., Keller and Frischknecht, 1966), where J_0 is the Bessel function of the first kind. To account for the earth, we specified two magnetic Hertz potentials, one for each medium (air, 0, and earth, 1),

$$\Pi^0 = (0, 0, \Pi_z^0) \quad z \ge 0 \tag{A-10}$$

and

 $\mathbf{\Pi}^1 = (0, 0, \Pi_z^1) \quad z \le 0, \tag{A-11}$

where

$$\Pi_{z}^{0} = M \frac{e^{-ik_{0}R}}{R} + M\Psi_{0}, \qquad (A-12)$$

$$\Pi_z^1 = M \Psi_1, \tag{A-13}$$

 $k_1 = (\omega^2 \mu_0 \varepsilon_1 - i\omega \mu_0 \sigma_1)^{1/2}$, and Ψ_0 and Ψ_1 are the secondary fieldexcitation functions. Following the approach of, for example, Wait (1951) and applying the necessary boundary conditions at the interface $(E_{\phi}^0 = E_{\phi}^1, H_{\rho}^0 = H_{\rho}^1)$, the Hertz potential components are

$$\Pi_{z}^{0} = M \int_{0}^{\infty} \left[\frac{\lambda}{u_{0}} e^{-u_{0}|z-h|} + \frac{\lambda(u_{0}-u_{1})}{u_{0}(u_{0}+u_{1})} e^{-u_{0}(z+h)} \right]$$
$$\times J_{0}(\lambda \rho_{d}) d\lambda \quad z \ge 0 \tag{A-14}$$

and

$$\Pi_{z}^{1} = M \int_{0}^{\infty} \frac{2\lambda}{u_{0} + u_{1}} e^{-u_{0}h} e^{u_{1}z} J_{0}(\lambda \rho_{d}) d\lambda \quad z \le 0, \qquad (A-15)$$

where $u_0 = (\lambda^2 - k_0^2)^{1/2}$ and $u_1 = (\lambda^2 - k_1^2)^{1/2}$. The components of the magnetic field are found from equation A-3. Since the first term of the integrand in equation A-14 corresponds to the primary field contribution and is known from the previous section, the secondary field above the earth is, using equation A-3,

$$H_x^0 = M \frac{(x - x_d)}{\rho_d} \int_0^\infty \frac{u_0 - u_1}{u_0 + u_1} e^{-u_0(z + h)} J_1(\lambda \rho_d) u_0 \lambda d\lambda, \quad (A-16)$$

$$H_{y}^{0} = M \frac{(y - y_{d})}{\rho_{d}} \int_{0}^{\infty} \frac{u_{0} - u_{1}}{u_{0} + u_{1}} e^{-u_{0}(z + h)} J_{1}(\lambda \rho_{d}) u_{0} \lambda d\lambda, \quad (A-17)$$

and

$$H_{z}^{0} = M \int_{0}^{\infty} \frac{\lambda(u_{0} - u_{1})}{u_{0}(u_{0} + u_{1})} e^{-u_{0}(z+h)} J_{0}(\lambda \rho_{d}) \lambda^{2} d\lambda.$$
 (A-18)

Neglecting displacement currents, $k_0 \rightarrow 0$, $k_1 \rightarrow (-i\omega\mu_0\sigma_1)^{1/2}$, and $u_0 \rightarrow \lambda$. The primary fields then reduce to

$$H_x^0 = M \frac{(x - x_d)}{\rho_d} \int_0^\infty \frac{\lambda - u_1}{\lambda + u_1} e^{-\lambda(z + h)} J_1(\lambda \rho_d) \lambda^2 d\lambda, \quad (A-19)$$

$$H_{y}^{0} = M \frac{(y - y_{d})}{\rho_{d}} \int_{0}^{\infty} \frac{\lambda - u_{1}}{\lambda + u_{1}} e^{-\lambda(z + h)} J_{1}(\lambda \rho_{d}) \lambda^{2} d\lambda, \quad (A-20)$$

and

$$H_z^0 = M \int_0^\infty \frac{\lambda - u_1}{\lambda + u_1} e^{-\lambda(z+h)} J_0(\lambda \rho_d) \lambda^2 d\lambda. \quad (A-21)$$

APPENDIX B

VERTICAL MAGNETIC DIPOLE OVER A HOMOGENEOUS, CONDUCTING EARTH WITH A BURIED LINEAR CONDUCTOR

In what follows, the reader is referred to Figure 2. The core problem definition is unchanged from that in the previous appendix. The conductor is infinite in length and has radius *a*, conductivity σ_c , permeability μ_c , and wavenumber $k_c = (-i\omega\mu_c\sigma_c)^{1/2}$. Subscript "*c*" will refer to the conductor.

POTENTIALS ARISING FROM THE PRIMARY EXCITATION

The potential resulting from the dipole source was handled in a manner similar to that in the previous appendix, but was expressed in a different form to enable matching with the conductor field. As before, a single magnetic Hertz vector having only a vertical component in the *z*-direction was used. Equation A-2 was rewritten using an integral form of the potential as (Wait, 1970)

$$\Pi_{z}^{d} = \frac{IA}{4\pi^{2}} \int_{-\infty}^{\infty} e^{-i\beta(y-y_{d})} K_{0} [(\beta^{2} - k_{0}^{2})^{1/2} \rho_{d}] d\beta, \quad (B-1)$$

where K_0 is the modified Bessel function of the second kind. As before, we specified two magnetic Hertz potentials, one for each medium (i.e., equations A-10 and A-11). Using an integral representation for K_0 (Wait, 1996) and handling the interfacial boundary conditions, the two Hertz-potential components in the air and the earth are

$$\Pi_{z}^{0} = \frac{IA}{8\pi^{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{-i\lambda(x-x_{d})}e^{-i\beta(y-y_{d})}}{u_{0}} [e^{-u_{0}|z-h|} + R_{d}e^{-u_{0}(z+h)}]d\lambda d\beta \quad z \ge 0$$
(B-2)

and

$$\Pi_{z}^{1} = \frac{IA}{8\pi^{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{-i\lambda(x-x_{d})}e^{-i\beta(y-y_{d})}}{u_{0}} T_{d}e^{-u_{0}h}e^{u_{1}z}d\lambda d\beta$$
$$z \le 0, \tag{B-3}$$

where $u_0 = (\beta^2 + \lambda^2 - k_0^2)^{1/2}, u_1 = (\beta^2 + \lambda^2 - k_1^2)^{1/2},$

$$R_d = \frac{u_0 - u_1}{u_0 + u_1},\tag{B-4}$$

and

$$T_d = \frac{2u_0}{u_0 + u_1}.$$
 (B-5)

The potential in the earth determines the driving electric field at the subsurface conductor that will induce the current in the conductor. We considered the situation where the conductor radius is much smaller than the operating wavelength in the earth and the burial depth. As a result, the thin-wire approximation can be made and only the axial current in the *y*-direction is significant. Therefore, the *y*-component of the electric field in the earth resulting from the source dipole was obtained from (Stratton, 1941)

$$\mathbf{E}^d = -i\omega\mu_0 \nabla \times \mathbf{\Pi}^1, \tag{B-6}$$

which yields (using the superscript "*d*" to indicate that this is the field resulting from the dipole source),

$$E_{y}^{d} = \frac{\mu_{0}\omega IA}{4\pi^{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\lambda}{u_{0} + u_{1}}$$
$$\times e^{-i\lambda(x - x_{d})} e^{-i\beta(y - y_{d})} e^{-u_{0}h} e^{u_{1}z} d\lambda d\beta \quad z \le 0.$$
(B-7)

Introducing the Fourier transform pair,

$$\chi(y) = \int_{-\infty}^{\infty} \hat{\chi}(\beta) e^{-i\beta y} d\beta$$
 (B-8)

and

$$\hat{\chi}(\beta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \chi(y) e^{i\beta y} dy, \qquad (B-9)$$

we identify

$$\hat{E}_{y}^{d}(\beta) = \frac{\mu_{0}\omega IA}{4\pi^{2}} \int_{-\infty}^{\infty} \frac{\lambda}{u_{0}+u_{1}} e^{-i\lambda(x-x_{d})} e^{-u_{0}h} e^{u_{1}z} e^{i\beta y_{d}} d\lambda$$
(B-10)

as the transform of the electric field resulting from the dipole source.

POTENTIALS RESULTING FROM THE CONDUCTOR

We began by considering the fields produced by the subsurface conductor (an infinite line of current) in a whole-space. To solve the desired boundary value problem, these fields can be derived from a linear combination of electric and magnetic Hertz vectors, each having only a *y*-component (Tsubota and Wait, 1980). Following Wait (1977), the electric Hertz vector was written as

$$\Lambda_{y}^{c} = \frac{1}{2\pi\sigma_{1}^{\prime}} \int_{-\infty}^{\infty} \hat{I}(\beta) K_{0} [(\beta^{2} - k_{1}^{2})^{1/2} \rho_{c}] e^{-i\beta y} d\beta,$$
(B-11)

where $\rho_c = [x^2 + (z + D)^2]^{1/2}$, $\sigma'_1 = \sigma_1 + i\varepsilon_1\omega$, and $\hat{I}(\beta)$ is the transform of the conductor current, I(y), which at this point is unknown. As before, we write the specific potentials for each medium ("c" will be implied) and apply the boundary conditions $E_x^0 = E_x^1, E_y^0 = E_y^1, H_x^0 = H_x^1$, and $H_y^0 = H_y^1$. The resulting electric Hertz vector components are

$$\Lambda_{y}^{0} = \frac{1}{4\pi\sigma_{1}'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{I}(\beta) \frac{e^{-i\lambda x}}{u_{1}} e^{-u_{1}D}$$
$$\times e^{-u_{0}z} T_{1}(\beta,\lambda) e^{-i\beta y} d\lambda d\beta \quad z \ge 0 \quad (B-12)$$

and

$$\Lambda_{y}^{1} = \frac{1}{4\pi\sigma_{1}'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{I}(\beta) \frac{e^{-i\lambda x}}{u_{1}} [e^{-u_{1}(z+D)} + R_{1}(\beta,\lambda)e^{-u_{1}(D-z)}]e^{-i\beta y}d\lambda d\beta \quad z \le 0, \quad (B-13)$$

where, using the expression found in Wait (1977) for R_1 ,

$$R_1 = -1 + \frac{2k_1^2 u_1}{k_1^2 - \beta^2} \left(\frac{1}{u_0 + u_1} - \frac{\beta^2}{k_0^2 u_1 + k_1^2 u_0} \right) \quad (\mathbf{B}-\mathbf{14})$$

and

$$T_1 = \frac{2k_1^2 u_1}{k_0^2 - \beta^2} \left(\frac{1}{u_0 + u_1} - \frac{\beta^2}{k_0^2 u_1 + k_1^2 u_0} \right).$$
(B-15)

The magnetic Hertz vector components take similar form,

$$\Pi_{y}^{0} = \frac{1}{4\pi\sigma_{1}'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{I}(\beta) \frac{e^{-i\lambda x}}{u_{1}} e^{-u_{1}D}$$
$$\times e^{-u_{0}z} T_{2}(\beta,\lambda) e^{-i\beta y} d\lambda d\beta \quad z \ge 0 \quad (B-16)$$

and

$$\Pi_{y}^{1} = \frac{1}{4\pi\sigma_{1}'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{l}(\beta) \frac{e^{-i\lambda x}}{u_{1}}$$
$$\times R_{2}(\beta,\lambda) e^{-u_{1}(D-z)} e^{-i\beta y} d\lambda d\beta \quad z \le 0, \qquad (B-17)$$

where

$$T_2 = \frac{i\beta\lambda(\kappa - 1)(1 + R_1)\kappa}{\mu_0\omega(u_0\kappa + u_1)},$$
 (B-18)

$$R_2 = \frac{i\beta\lambda(\kappa - 1)(1 + R_1)}{\mu_0\omega(u_0\kappa + u_1)},$$
 (B-19)

and $\kappa = (k_1^2 - \beta^2)/(k_0^2 - \beta^2)$.

THE CURRENT INDUCED IN THE CONDUCTOR

The conductor current and its transform were determined from the axial impedance boundary condition at the surface of the conductor. The general form of the impedance condition is given by (e.g., Hill and Wait, 1977; Wait, 1977)

$$\int_{-\infty}^{\infty} \hat{I}(\beta) Z_i(\beta) e^{-i\lambda(y-y_d)} d\beta = (E_y^c + E_y^d)_c, \quad (\mathbf{B}\text{-}20)$$

where $Z_i(\beta)$ is the impedance of the conductor per unit length (referred to as the internal impedance), and $(E_y^c)_c$ and $(E_y^d)_c$ are the axial electric fields resulting from the unknown conductor current I(y) and the dipole source, respectively, each evaluated at the conductor. We chose to evaluate the dipole source potential at the center of the conductor and the conductor potential at the surface of the conductor $(\rho_c = a)$. This is a valid simplification since the conductor radius is much smaller than both the operating wavelength in the earth and the conductor depth.

The internal impedance of the conductor is given by Wait (1977) for $|\beta^2| \ll |k_c^2|$ and $\sigma_c < \varepsilon_c \omega$ as

$$Z_i(\beta) \approx Z_i = \frac{1}{2\pi a} \left(\frac{i\mu_c \omega}{\sigma_c} \right)^{1/2} \frac{I_0(ik_c a)}{I_1(ik_c a)}, \qquad (B-21)$$

where I_0 and I_1 are modified Bessel functions of the first kind. Following Wait (1977), we have

$$\hat{I}(\beta) = \frac{(\hat{E}_{y}^{d})_{c}}{Z_{i} + Z_{e}(\beta)},$$
(B-22)

where $Z_e(\beta)$ is an "external impedance" resulting from the surrounding earth. It follows that

$$Z_e(\beta) = -\frac{(\hat{E}_y^c)_c}{\hat{I}(\beta)},\tag{B-23}$$

and from equation B-10, the transform of the electric field resulting from the dipole evaluated at the center of the conductor (x = 0, z = -D) is

$$(\hat{E}_{y}^{d})_{c} = \frac{\mu_{0}\omega IA}{4\pi^{2}} \int_{-\infty}^{\infty} \frac{\lambda}{u_{0} + u_{1}} e^{i\lambda x_{d}} e^{-u_{0}h} e^{-u_{1}D} e^{i\beta y_{d}} d\lambda. \quad (B-24)$$

The electric field in the earth resulting from the conductor becomes

$$E_{y}^{c} = \frac{k_{1}^{2} - \beta^{2}}{4 \pi \sigma_{1}'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{I}(\beta) \frac{e^{-i\lambda x}}{u_{1}} [e^{-u_{1}(z+D)} + R_{1}e^{-u_{1}(D-z)}]e^{-i\beta y} d\lambda d\beta, \qquad (B-25)$$

and the transform of the electric field resulting from the conductor is therefore

$$\hat{E}_{y}^{c} = \frac{k_{1}^{2} - \beta^{2}}{4\pi\sigma_{1}^{\prime}} \int_{-\infty}^{\infty} \hat{I}(\beta) \frac{e^{-i\lambda x}}{u_{1}} [e^{-u_{1}(z+D)} + R_{1}e^{-u_{1}(D-z)}]d\lambda.$$
(B-26)

From equation B-23, we have

1

$$Z_e(\beta) = \frac{\beta^2 - k_1^2}{4\pi\sigma_1'} \left[\int_{-\infty}^{\infty} \frac{e^{-i\lambda x}}{u_1} e^{-u_1(z+D)} d\lambda + \int_{-\infty}^{\infty} \frac{e^{-i\lambda x}}{u_1} R_1 e^{-u_1(D-z)} d\lambda \right]_c, \quad (B-27)$$

noting that both integrals must be evaluated at the conductor. Using the integral form of K_0 and the assumption that the conductor radius is small relative to all other problem dimensions, we have

$$Z_{e}(\beta) = \frac{\beta^{2} - k_{1}^{2}}{2\pi\sigma_{1}'} \Biggl\{ K_{0}[(\beta^{2} - k_{1}^{2})^{1/2}a] - K_{0}[2D(\beta^{2} - k_{1}^{2})^{1/2}] + \frac{k_{1}^{2}}{k_{1}^{2} - \beta^{2}} \int_{-\infty}^{\infty} \left(\frac{1}{u_{0} + u_{1}} - \frac{\beta^{2}}{k_{0}^{2}u_{1} + k_{1}^{2}u_{0}}\right)e^{-2u_{1}D}d\lambda \Biggr\}.$$
(B-28)

We can compute the transform of the conductor current from equation B-22, using B-21, B-24, and B-28.

THE MAGNETIC FIELD ABOVE THE SURFACE RESULTING FROM THE CONDUCTOR

The magnetic fields resulting from the conductor as measured by a receiver located at (x,y,z) with z > 0 are obtained from Stratton (1941)

$$\mathbf{H} = \boldsymbol{\sigma}_0' \nabla \times \boldsymbol{\Lambda}^0 + \nabla \nabla \cdot \boldsymbol{\Pi}^0 + k_0^2 \boldsymbol{\Pi}^0, \qquad (\mathbf{B}\text{-}\mathbf{29})$$

where $\sigma'_0 = \sigma_0 + i\varepsilon_0 \omega$ is the complex conductivity of the air. The general expressions are

$$H_x^c = \frac{1}{2\pi} \int_{-\infty} \int_{-\infty} \hat{l}(\beta) F_x e^{-i(\lambda x + \beta y)} e^{-(u_1 D + u_0 z)} d\lambda d\beta, \quad (B-30)$$

$$H_{y}^{c} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{I}(\beta) F_{y} e^{-i(\lambda x + \beta y)} e^{-(u_{1}D + u_{0}z)} d\lambda d\beta, \quad (B-31)$$

and

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$$H_z^c = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{I}(\beta) F_z e^{-i(\lambda x + \beta y)} e^{-(u_1 D + u_0 z)} d\lambda d\beta, \quad (B-32)$$

where

$$F_{x} = \frac{\beta^{2} [\sigma_{0}' u_{0}(u_{0} + u_{1}) + \lambda^{2} (\sigma_{1}' - \sigma_{0}')] + i\mu\omega\sigma_{0}' u_{0}(\sigma_{1}' u_{0} + \sigma_{0}' u_{1})}{(\beta^{2} + i\mu\omega\sigma_{0}')(u_{0} + u_{1})(u_{0}\sigma_{1}' + u_{1}\sigma_{0}')},$$
(B-33)

$$F_{y} = \frac{\beta \lambda (\sigma_{1}' - \sigma_{0}')}{(u_{0} + u_{1})(u_{0}\sigma_{1}' + u_{1}\sigma_{0}')},$$
 (B-34)

and

$$F_z = -\frac{i\lambda}{u_0 + u_1}.$$
 (B-35)

Neglecting all displacement currents, the field expressions reduce to

$$H_{j}^{c} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{I}(\beta) F_{j} \frac{\lambda}{u_{0}(u_{0}+u_{1})}$$
$$\times e^{-i(\lambda x+\beta y)} e^{-(u_{1}D+u_{0}z)} d\lambda d\beta \qquad (B-36)$$

where

$$F_{j} = \begin{cases} \lambda & j = x \\ \beta & j = y \\ -iu_{0} & j = z \end{cases}$$
(B-37)

We have compared our results for the vertical magnetic field resulting from the conductor to those of Tsubota (1979) for a few select scenarios and the agreement is very good.

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